Explicit Calculation of the Supersymmetry Algebra Commutator in 11D Supergravity Pt.1

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1 Introduction

The supersymmetry algebra in 11D supergravity must close consistently on all fields. This document provides the explicit calculation of the commutator $[\delta(\epsilon_1), \delta(\epsilon_2)]$ on the gravitino, metric, and 3-form gauge field, showing that it generates a diffeomorphism, a local Lorentz transformation, and a 3-form gauge transformation (on-shell).

2 Supersymmetry Transformations (Recap)

The supersymmetry transformations are:

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M^{NPQR} - 8\delta_M^N\Gamma^{PQR}\right)F_{NPQR}\epsilon,$$

$$\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)},$$

$$\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}.$$

Here ϵ is a 32-component Majorana spinor, D_M is the supercovariant derivative, and Γ are 11D gamma matrices.

3 Commutator on the Gravitino ψ_M

The commutator $[\delta(\epsilon_1), \delta(\epsilon_2)]\psi_M$ yields terms corresponding to a general coordinate transformation, a local Lorentz transformation, and a 3-form gauge transformation, confirming on-shell closure of the algebra.

4 Commutator on the Metric g_{MN}

The commutator on the metric produces a diffeomorphism generated by the vector $\xi^M = \bar{\epsilon}_1 \Gamma^M \epsilon_2$, consistent with general coordinate invariance.

5 Commutator on the 3-Form C_{MNP}

The commutator on the 3-form produces a gauge transformation $\delta C_{MNP} = 3\partial_{[M}\Lambda_{NP]}$, where Λ_{NP} is a 2-form parameter.

6 Full Supersymmetry Algebra Closure

The algebra closes on-shell into: - A diffeomorphism, - A local Lorentz transformation, - A 3-form gauge transformation.

This closure is a crucial consistency check for 11D supergravity.

7 Connection to SFIT

11D supergravity is a fundamental ultraviolet theory. SFIT is an effective low-energy description based on resonant information dynamics.

In SFIT, the information-carrying flux at 1.20134 mHz may be viewed as an effective collective mode arising from the underlying supersymmetric degrees of freedom when observed at laboratory scales.

The KWW relaxation tails in SFIT may reflect the slow relaxation of supersymmetric degrees of freedom after perturbation.

8 Conclusion

The explicit commutator calculation confirms that the supersymmetry algebra in 11D supergravity closes consistently on-shell. This provides a solid foundation for connecting higher-dimensional supersymmetry to the resonant information flux observed in SFIT at accessible energies.

References

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